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Role of the effective Hilbert-space size of the reservoir for the decoherence process

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We show that an environment composed by N bosons coupled through cross-Kerr interaction to an oscillator of interest can be effective at destroying quantum coherences at short times and around the revival times even if $N=1$. It is analytically shown for this model that the effective Hilbert-space size is a relevant parameter for decoherence process. Based on numerical results, we investigate the long time dynamics and the classical limit. Since we are dealing with a phase reservoir, the model does not describe dissipation.

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I. INTRODUCTION

There are situations where the direct application of the Schröedinger equation leads to unsatisfactory results: a well known example is dissipation. Dissipation can be phenomenologically modeled by two main approaches, the complex Hamiltonian [1,2] and open system [3–6]; it is also possible to quantize a non-Hamiltonian system [7], but this approach may be reduced to the open system one. The open system approach is based on the fact that when we are dealing with a real physical system there is a huge number of particles involved. Due to the impossibility of computing the dynamics of each particle, it is common to study the statistics of their effects over the system of interest by considering this system coupled to an environment, frequently modeled as an infinity degree of freedom reservoir. While dissipation may be described in Classical Mechanics, a strictly quantum process is also induced by the environment: decoherence [8,9], which occurs independently from dissipation [10,11], and corresponds to the progressive disappearing of quantum correlations. Decoherence may be quantified in terms of entropy, or linear entropy [12,13]: if the linear entropy of a single system is zero, we can say that it did not lose quantum coherence. This process can be summarized as Omnes's definition [14]: *"Decoherence is a dynamic effect through which the states of the environment associated with different collective states become rapidly orthogonal. It comes from a loss of local phase correlation between the corresponding wave functions of the environment, which is due to the interaction between the collective system and the environment, also responsible for dissipation. It depends essentially upon the fact that the environment has a very large number of degrees of freedom."* Decoherence has been experimentally observed [15].

One problem in the open system approach is the difficulty of modeling the bath by first principles. For many practical situations the environment has been chosen as a collection of harmonic oscillators [4,8,9,12,14,16–18] and its phenomenological success has been proven for some cases [19–21]. In the context of the classical limit problem, there is a strong debate on the necessity of using an environment in order to recover classical dynamics from quantum mechanics [22–25]. Indeed, it was demonstrated that diffusion can produce only quantum coherence attenuation, being not able to eliminate it [12]; thus we are forced to include a finite experimental resolution to reach the classical limit [26].

Although the environment is usually modeled by using many degrees of freedom, we can find examples from the quantum chaos of one degree of freedom systems that produces decoherencelike behavior: in Ref. [27], N two-level atoms are mapped in one multilevel system that is capable of inducing decoherence in a bosonic system coupled to it (for chaotic initial conditions of the classical counterpart, the loss of coherence is faster); one inverted oscillator, which approximates the exponential sensitivity of chaotic systems, is used in Ref. [28] as a quantum environment that can be more effective at destroying quantum coherences than usual many degrees of freedom environments; a single particle deterministic conservative chaotic environment described by the quantum kicked rotator model was shown to be able to reproduce the effects of a pure dephasing many-body bath [29,30]; an analytical and numerical investigation on the equivalence between a quantum chaotic system and a many-body system as a decohering environment was recently performed [31]. In the present contribution, we show that decoherencelike behavior may be induced by one degree of freedom environment without chaos. As a model, we use a quartic oscillator (the system) coupled through cross-Kerr interaction to N bosons (the environment). This model is analytically solvable and its classical limit is characterized by a unique time [11,32], while in general there are many time scales [33]; also, it has experimental interest: the model was used to study Bose-Einstein condensate [34] and Kerr-like medium [35]. We must point out that the small system that plays the environment will not relax to a unique thermal equilibrium state. Also, the considered model does not describe dissipation.

The interaction Hamiltonians considered here commute with the whole Hamiltonian, characterizing a phase reservoir which produces adiabatic coherence loss. There are many

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important investigations in the literature using such reservoirs [10,11,32,36–46]. Some examples: in Ref. [36], it is shown that the phase-damping reservoir can be used to model electronic dynamics in organic charge-transfer salts; decoherence process induced by an adiabatic reservoir is studied in Refs. [37–39], where it is shown that this process is controlled by the spectral properties of the interaction; it is found in Ref. [40] that decoherence is independent of the temperature for an adiabatic reservoir modeled by an infinite number of two-level systems; the impact of a phase reservoir on the efficiency of quantum algorithms is analyzed in Ref. [41]; in Refs. [43,44], it is shown that the stationary state of a system coupled to an adiabatic environment depends on the initial state of the system; the effects of a phase-damping reservoir on the dynamical Casimir effect was investigated in Ref. [45]; in Ref. [11], it is shown that nondissipative decoherence can be suitably contemplated within the environment-induced decoherence approach; a phase reservoir with a Gaussian frequency distribution is analyzed in Ref. [32], where the author concludes that diffusive decoherence is necessary to establish quantum-classical correspondence. In our contribution, we studied the equivalence between phase reservoirs with different numbers of degrees of freedom, and found situations where the classical limit may be induced even by a reservoir with one oscillator.

As pointed out in Ref. [28], the total amount of entropy that can be produced by an environment initially in a pure state is limited by the size of its Hilbert space. A natural question concerns how the spreading of the state of system plus environment in the Hilbert-space effects the entropy production. Since there is not a single basis to describe the dynamics, we must choose the basis to measure this spreading. In the present work, we choose the interaction Hamiltonian eigenstates basis, and show that, for the present model, the variances of the states of the system and the environment in this basis determine different characteristic times related to decoherence. Since the size of the Hilbert space occupied in such a basis is infinite for many important states, the link of decoherence process and Hilbert-space size is not immediate. In order to perform such a link, we studied decoherence process for the environment in the Pegg-Barnett phase state [48], which is distributed uniformly in a finite Hilbert space, and defined the *effective Hilbert-space size* as the Hilbert-space size of the phase state that generates purity loss equivalently as the other particular environmental states considered. Our results, obtained for particular cases, confirm the intuition that the loss of quantum coherence induced by the environment tends to be more complete if the environmental Hilbert space is more effectively occupied. We believe analogous conclusions may be found for other systems.

The effective Hilbert-space size is a measure of the capacity of the environment to produce entropy at the system of interest. For the model studied here, it depends on the environmental state and the interaction Hamiltonian [see Eqs. (11) and (17)] but does not depend on the state of the system of interest, whose effects for the decoherence times are computed separately. In the present stage of our investigation, we cannot affirm that such separate computation is always possible. In the cases where this is possible, the effective Hilbert-space size may be used to compare environments with different numbers of degrees of freedom, independently of the state of the system they are coupled to.

The initial entropy of the environment does not play a central role for decoherence process in our model. Calculations considering environmental states with different entropies may lead to the same effective Hilbert-space size. These states, which may be a (pure) coherent state and a thermal state, generate equivalent decoherence characteristic times. It is important to stress that there are investigations where the environmental initial entropy plays a central role. As an example, let us consider the analysis performed in Ref. [47], where the environment is seen as a source of information about the system of interest. The capacity of the environment to store information is reduced by its initial von Neumann entropy.

II. PHASE RESERVOIR MODEL

Let us consider a general phase reservoir Hamiltonian

$$H = H_1 + H_2 + H_{12}, \tag{1}$$

where H_1 stands for the system of interest, H_2 concerns the reservoir and H_{12} is the interaction term. The phase reservoir is characterized by the commutation relations

$$[H,H_1] = [H,H_2] = [H,H_{12}] = 0. \tag{2}$$

Taking $|\phi_n\rangle$ and $|\psi_k\rangle$ as eigenvectors of H_1 and H_2, respectively, relation (2) permits us to write $H|\phi_n\rangle|\psi_k\rangle = E_{n,k}|\phi_n\rangle|\psi_k\rangle$. If H_2 refers to M independent subsystems, we may assume

$$H_2 = \sum_{l=1}^{M} H_2^l, \quad H_{12} = \sum_{l=1}^{M} H_{12}^l, \tag{3}$$

with $[H_2^l, H_2^k] = [H_{12}^l, H_{12}^k] = 0$. The eigenvectors and eigenvalues will be given by $H_2^l|\psi_{r_l}^l\rangle = E_{r_l}^l|\psi_{r_l}^l\rangle$ and $H_{12}^l|\phi_n\rangle|\psi_{r_l}^l\rangle = E_{n,r_l}^{1,l}|\phi_n\rangle|\psi_{r_l}^l\rangle$. In order to quantify the entanglement between the system of interest and the environment, we may use the linear entropy $\delta(t)$. If $\rho(t)$ is the density operator for the whole system and $\rho_1(t)$ is its trace over the environmental variables, the linear entropy is calculated as $\delta(t) = 1 - \mathrm{Tr}\{[\rho_1(t)]^2\}$. For the generic initial state

$$\rho(0) = \sum_{v,w} B_{v,w}|\phi_v\rangle\langle\phi_w| \prod_{l=1}^{M} \left\{ \sum_{r_l,s_l} A_{r_l,s_l}^l |\psi_{r_l}^l\rangle\langle\psi_{s_l}^l| \right\}, \tag{4}$$

we find

$$\delta(t) = 1 - \prod_{l=1}^{N} \left\{ \sum_{r_l,s_l} \sum_{v,w} A_{r_l,r_l}^l A_{s_l,s_l}^l B_{v,w} B_{w,v} \exp\left[-\frac{it}{\hbar}\left(E_{v,r_l}^{1,l} - E_{w,r_l}^{1,l} - E_{v,s_l}^{1,l} + E_{w,s_l}^{1,l}\right)\right]\right\}. \tag{5}$$

Due to commutation relation (2), $\delta(t)$ depends only on the interaction part of the whole Hamiltonian, as may be seen in the equation above.

In order to analyze a specific case, we now assume

$$H_{12} = \sum_{l} \hbar\lambda_l (N_1)^x (N_2^l)^y, \tag{6}$$

where $N_1|\phi_v\rangle = v|\phi_v\rangle$ and $N_2^l|\psi^l_{r_l}\rangle = r_l|\psi^l_{r_l}\rangle$ (v and r_l are integer numbers), leading to

$$\delta(t) = 1 - \prod_{l=1}^{M} \left\{ \sum_{r_l,s_l} \sum_{v,w} A^l_{r_l,r_l} A^l_{s_l,s_l} B_{v,w} B_{w,v} \exp\{-it\lambda_l[(v^x - w^x) \times (r_l^y - s_l^y)]\} \right\}. \tag{7}$$

The linear entropy dynamics is usually characterized by the decoherence time [13] and the revival time [11,33]. Performing the expansion $\delta(t) \approx \delta(0) + \delta_1 t + \delta_2 t^2 + O(t^3)$, it is easy to see that $\delta_1 = 0$. Thus, the decoherence time is defined as $t_D = 1/\sqrt{\delta_2}$ and, for the system of interest initially in a pure state ($\delta(0) = 0$), the explicit expression for this time reads

$$t_D = \frac{1}{\Delta_1 \sqrt{2\sum_{l=1}^{M} (\lambda_l \Delta_2^l)^2}}, \tag{8}$$

where $(\Delta_1)^2$ is the variance of operator $(N_1)^x$ and $(\Delta_2^l)^2$ is the variance of operator $(N_2^l)^y$ calculated for $\rho(0)$. Decoherence time (8) depends solely on the coupling Hamiltonian and the initial state of system plus environment. The system of interest recovers its initial purity at the revival time

$$t_R = \frac{2\pi}{\Lambda} s, \tag{9}$$

where $s \in \mathbb{N}$, $\Lambda = 2\pi/T_\Lambda$, and T_Λ is the least common multiple of the periods in the set $\{T_l = 2\pi/\lambda_l\}$, supposing that it exists. In this case, we may define the revival lifetime τ_R as the time interval when the recovered purity can be observed. By expanding $\delta(t)$ around t_R, we obtain $\tau_R = 2t_D$, which may be written as

$$\tau_R = \frac{\sqrt{2}}{\Lambda \Delta_1 \Delta_2}, \tag{10}$$

where

$$\Delta_2 = \sqrt{\sum_{l=1}^{M} (k_l \Delta_2^l)^2} \tag{11}$$

and $k_l = \lambda_l/\Lambda$.

The expressions for t_D and τ_R indicate that the loss of quantum coherence around the times $t=0$ and $t=t_R$ depends crucially on the product $\Delta_1\Delta_2$ compared to $1/\Lambda$. Thus, as concerns the dynamics of $\delta(t)$ at these intervals, a reservoir composed of M systems is phenomenologically equivalent to one degree of freedom with the same effective coupling constant Λ and state variance $(\Delta_2)^2$. Since these revivals are quantum signatures [12,33], the above expressions of Eqs. (9) and (10) suggest a classical limit as $\Delta_2 \to \infty$ and $\Lambda \to 0$ with $\Lambda\Delta_2 \to \infty$.

III. QUARTIC OSCILLATOR

We get some insight by investigating the nonlinear oscillator Hamiltonian,

$$H = \hbar\omega a^\dagger a + \hbar g (a^\dagger a)^2 + \sum_{i=1}^{M} \hbar\Omega b_i^\dagger b_i + \sum_{j=1}^{M} \hbar\lambda a^\dagger a b_j^\dagger b_j, \tag{12}$$

where $a^\dagger$ (a) and $b_i^\dagger$ (b_i) are creation (annihilation) bosonic operators. This Hamiltonian fulfils the conditions discussed above with $x=y=1$ and has the same eigenstates of the harmonic oscillator. From now on we assume the initial state

$$\rho(0) = \frac{1}{2}\{|0\rangle\langle 0| + |1\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 1|\} \otimes \prod_{k=0}^{M} r_k(0), \tag{13}$$

where $|0\rangle$ and $|1\rangle$ are Fock states concerning the system of interest and $r_k(0)$ refers to the state of the kth environmental oscillator. Considering the reservoir in thermal equilibrium, i.e.,

$$r_k(0) = \left[1 - \exp\left(-\frac{\hbar\Omega}{k_B T}\right)\right] \sum_{n_k=0}^{\infty} \exp\left(-\frac{\hbar\Omega}{k_B T} n_k\right) |n_k\rangle\langle n_k|, \tag{14}$$

where $\{|n_k\rangle\}$ are Fock states, k_B is Boltzmann's constant and T the absolute temperature, we obtain

$$\delta(t) = \frac{1}{2}\left\{1 - \left[\frac{\left[1 - \exp\left(-\frac{\hbar\Omega}{k_B T}\right)\right]^2}{1 + \exp\left(-\frac{2\hbar\Omega}{k_B T}\right) - 2\exp\left(-\frac{\hbar\Omega}{k_B T}\right)\cos(\lambda t)}\right]^M\right\}, \tag{15}$$

that explicitly shows that the system recovers purity at $t=t_R$ even for a large number of environmental oscillators. We can observe in Fig. 1 that the linear entropy around $t=0$ and $t=t_R$ grows faster as the total number of oscillators is increased, as expected, but also for higher temperatures. In the examples shown in Fig. 1, a single oscillator at high temperature (high Δ_2) and M oscillators with low energy produce similar entropy enhancements. In order to stress the fact that many features of such a dynamics do not depend on the specific reservoir initial state, but only on Δ_2, we plotted in Fig. 2 the evolution of $\delta(t)$ for the reservoir initially in Pegg-Barnett phase state [48] $|\varphi_m\rangle(r)$, defined as

$$|\varphi_m\rangle(r) = \frac{1}{\sqrt{r+1}} \sum_{n=0}^{r} e^{in\varphi_m} |n\rangle, \tag{16}$$

where $\varphi_m = \frac{2\pi m}{r+1}$, $m = 0, 1, \ldots, r$.

The Pegg-Barnett phase state has a uniform distribution over the lowest number states which form a truncated $(r+1)$-dimensional Hilbert space, leading to $\Delta_2 = [r(r+2)/12]^{1/2}$. For any reservoir initial state, we can find a phase state with approximately the same Δ_2, generating approximately the same characteristic times t_D and τ_R. We define the *effective Hilbert-space size* as the truncated Hilbert-space size of such an equivalent phase state,

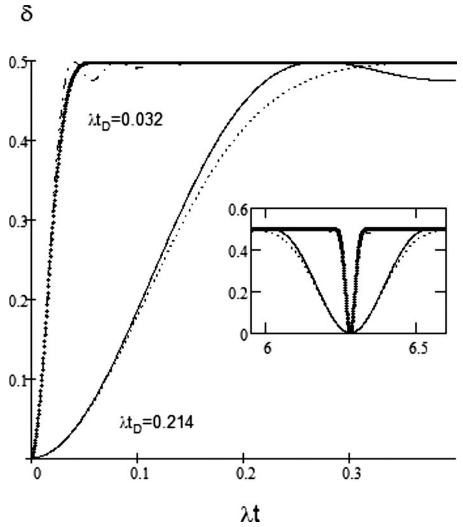

FIG. 1. Linear entropy for thermal initial environmental state. Thick dotted line: $M=201$, $\Delta_2=3.16$, and $\lambda t_D \approx 0.032$. Dash-dotted line: $M=1$, $\Delta_2=44.83$, and $\lambda t_D \approx 0.032$. Full line: $M=1$, $\Delta_2=6.61$, and $\lambda t_D \approx 0.214$. Thin dotted line: $M=15$, $\Delta_2=1.71$, and $\lambda t_D \approx 0.214$. The insert shows the linear entropy around t_R. The horizontal axis corresponds to λt. For all plots we used Hamiltonian (12) with $g=1$ and $\hbar=1$, and initial state (13).

$$Hs = \sqrt{1 + 12(\Delta_2)^2}. \tag{17}$$

If $y \neq 1$, we can use the same receipt: for each specific N_2^y variance, there is an equivalent r which defines Hs.

In Fig. 3, we plot the $M=1$, $\Delta_2=44.83$ case of Fig. 2 for the time interval concerning $2000t_D$: this shows several revivals. The characteristic times t_D and τ_R give information about the dynamics near the revivals; far from the revivals, the linear entropy is always near its maximum value 0.5. Thus, since t_D and τ_R depend only on Δ_2, or equivalently on Hs, at least for the cases investigated here and the period shown, the dynamics of $\delta(t)$ depends mainly on Hs.

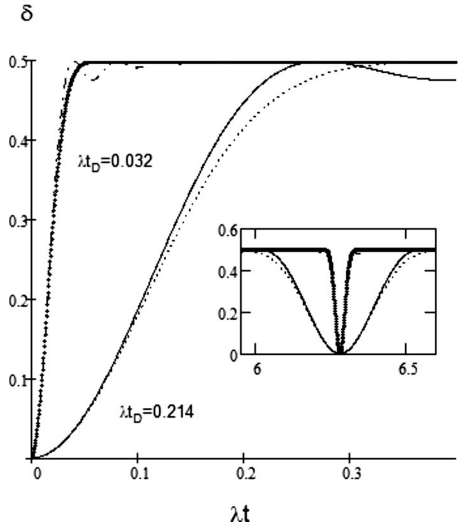

FIG. 2. Linear entropy for the environment initially in phase state with $m=0$. Other parameters and descriptions are the same as in Fig. 1.

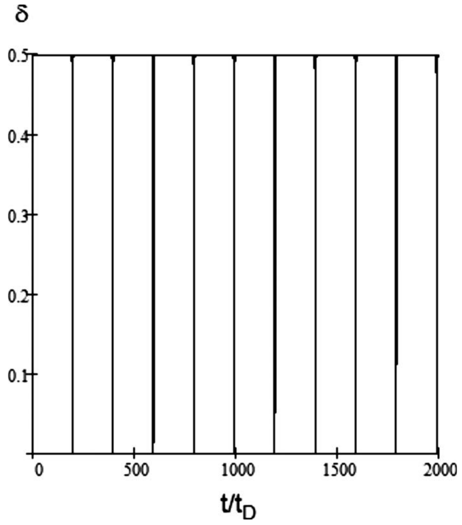

FIG. 3. Linear entropy for the environment initially in phase state with $m=0$, $M=1$, and $\lambda t_D \approx 0.032$. The horizontal axis corresponds to t/t_D. We used Hamiltonian (12) with $g=1$ and $\hbar=1$, and initial state (13).

Now let us say a word about quantum uncertainty and reversibility. The above results show that the revival lifetime τ_R goes to zero in the limit $\Delta_2 \to \infty$. An important point that must be stressed: this limit can be obtained even for an environment with $M=1$, i.e., a one system reservoir in infinity temperature. In fact we have not infinity temperature in laboratories; however, measuring this revival for high temperatures demands high time precision. The time uncertainty is related to the system energy uncertainty by Heisenberg uncertainty principle as $\Delta t \Delta E \geq \hbar/2$. Considering our initial state (13), and as we are interested in revival dynamics, we get $\Delta t \geq 1/(g+\omega)$. For $\Delta t \gg \tau_R$, or $\sqrt{2}/(\Delta_1\Delta_2) \ll \lambda/(g+\omega)$, we would have a vanishing probability of observing the revival. The Heisenberg uncertainty relation gives a fundamental limitation in experimental resolution; in real experiments, time uncertainty is always greater [49]. This suggests a classical limit based on coarse-grained measurement [24,25].

In Fig. 4, we plot the linear entropy evolution for initial phase states and a new coupling term:

$$H_{12} = \sum_{j=1}^{M} \hbar \lambda a^\dagger a \sqrt{b_j^\dagger b_j}. \tag{18}$$

Again we see the important role played by λ and Δ_2 for decoherence process at short times. In Fig. 5, we see that relevant revivals do not occur in the interval of $2000t_D$, suggesting that another way for revival vanishing is to assume that x or y in Eq. (6) are not integers.

In order to focus on the quantum-classical transition problem, we now consider the recurrence time t_r, defined as the time when the system of interest recovers its initial state: $\rho_1(t_r)=\rho_1(0)$. It was shown in Ref. [12] that the quantum-classical break time for the quartic oscillator is $t_r/2$. In Refs. [12,50], the quantum behavior was partially suppressed when the quartic oscillator was coupled to a diffusive reservoir.

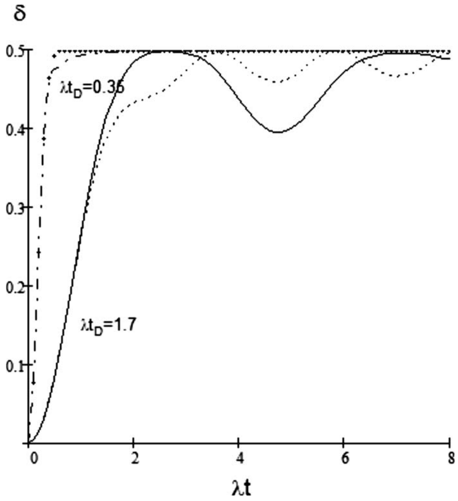

FIG. 4. Linear entropy for the environment initially in phase state with $m=0$. Thick dotted line: $r=10$, $M=20$, and $\lambda t_D \approx 0.35$. Dash-dotted line: $r=289$, $M=1$, and $\lambda t_D \approx 0.35$. Full line: $r=2$, $M=2$, and $\lambda t_D \approx 1.7$. Thin dotted line: $r=8$, $M=1$, and $\lambda t_D \approx 1.7$. The horizontal axis corresponds to λt. We used interaction Hamiltonian (18) with $g=1$ and $\hbar=1$, and initial state (13).

Here, we will analyze the dynamics of the quartic oscillator coupled to the phase reservoir, with the Hamiltonian given by Eq. (12). Considering all environmental bosons initially in the same state, we get, for $\lambda=0$, $t_r=2\pi n/g$, $n \in \mathbb{N}$. Using the fact that for $\lambda \neq 0$ the state of system plus environment is factorized at $t_\lambda=2\pi m/\lambda$, $m \in \mathbb{N}$, we calculate the recurrence time for $g>\lambda$ as $t_r=2\pi s/g$, where $s \in S$, with $S=\{p \in \mathbb{N} | \frac{\lambda}{g} p \in \mathbb{N}\}$.

Analogously as in Ref. [12], we will analyze the dynamics of the expectation value of the position operator $\langle X(t)\rangle = \{\mathrm{Tr}[\rho_1(t)X]\}$. For generic initial state (4) we find, after some straightforward algebra,

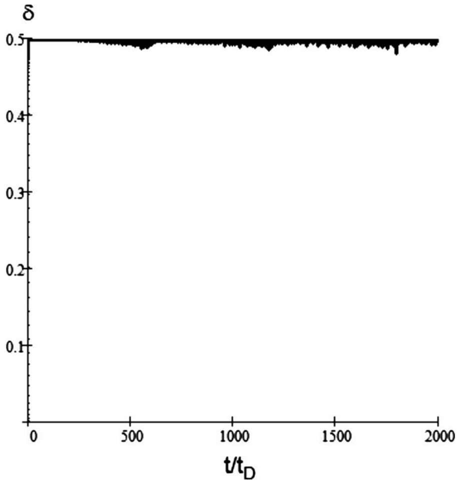

FIG. 5. Linear entropy for the environment initially in phase state with $m=0$, $M=1$, and $\lambda t_D \approx 0.35$. The horizontal axis corresponds to t/t_D. We used interaction Hamiltonian (18) with $g=1$ and $\hbar=1$, and initial state (13).

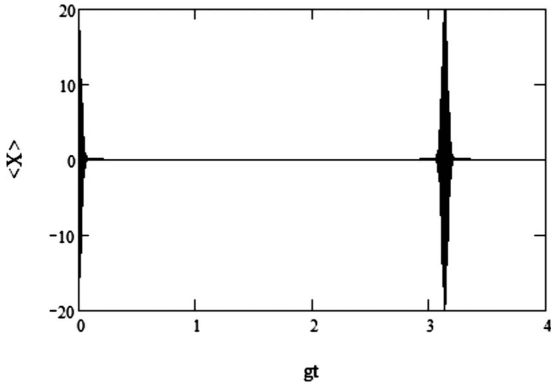

FIG. 6. Position expectation value for $\lambda=0$ and $\alpha=20$. The horizontal axis corresponds to gt. We used Hamiltonian (12) with $\omega=100g$ and $\hbar=1$, and initial state (13).

$$\langle X(t)\rangle = \mathrm{Re}\left\{ \prod_{l=1}^{M} \left[\sum_{rl} \sum_{v} \sqrt{v-1} A^{l}_{rl,rl} B_{v,v-1} e^{-it/\hbar[\Gamma(v,v-1,r_l)]} \right] \right\}, \tag{19}$$

where

$$\Gamma(v,w,r_l) = \hbar\omega + \hbar g(2v-1) + \hbar\lambda r_l, \tag{20}$$

and we considered a system of units where $X=\mathrm{Re}\{a\}$. If the system of interest is initially at the coherent state $|\alpha\rangle$, we can write $\langle X(t)\rangle$ in the form

$$\langle X(t)\rangle = \mathrm{Re}\{\langle a_q(t)\rangle \chi(t)\}, \tag{21}$$

where

$$\langle a_q(t)\rangle = \alpha e^{-i(\omega+g)t} e^{-|\alpha|^2[1-\exp(-2itg)]} \tag{22}$$

gives the evolution of $\langle X(t)\rangle$ in the absence of the reservoir and

$$\chi(t) = \prod_{l=1}^{M} \left\{ \sum_{rl} A^{l}_{rl,rl} \exp\{-it[\lambda r_l]\} \right\}. \tag{23}$$

In Figs. 6 and 7, we observe the first revival of $\langle X(t)\rangle$ occurring at the time $t_r/2=100\pi\omega^{-1}$ for an isolated system of interest ($\lambda=0$); indeed, $\langle X(t+t_r/2)\rangle = -\langle X(t)\rangle$. If we couple

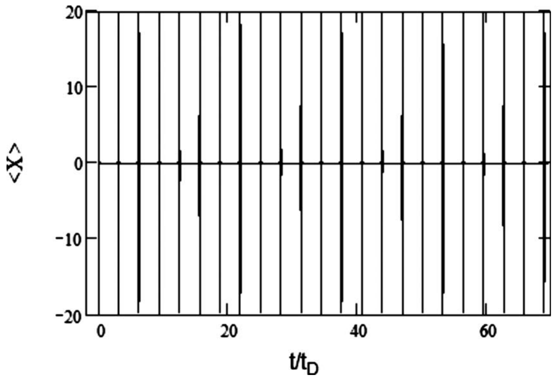

FIG. 7. Position expectation value for longer time. The parameters and descriptions are the same as in Fig. 6.

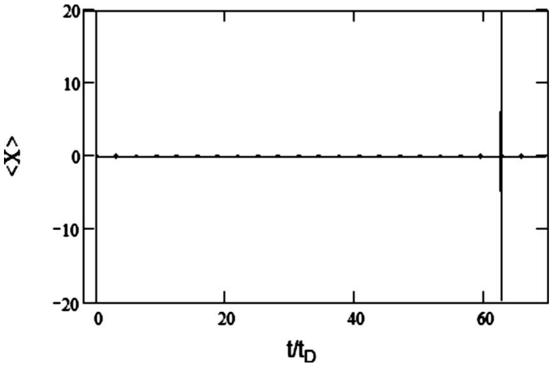

FIG. 8. Position expectation value for $\lambda = 0.1g$, $\alpha = 20$, $M = 1$, and $k_bT = 245.4$. The horizontal axis corresponds to gt. We used Hamiltonian (12) with $\omega = 100g$ and $\hbar = 1$, and initial state (13).

the system of interest with the one degree of freedom phase reservoir, the revivals may be retarded: in Fig. 8, the first revival is found at the first recurrence time $t_r = 2000\pi\omega^{-1}$.

We can calculate the revival lifetime for the position operator mean value, analogously as in the case of linear entropy. Observing that the revivals occur when $\langle a_q(t)\rangle\chi(t) \approx \alpha e^{-i(\omega+g)t}$ and writing this term as $\langle a_q(t)\rangle\chi(t) \approx \alpha e^{-i(\omega+g)t}[1 + D_1 t + O(t^2)]$, we define the position revival lifetime as $t_R = 1/|D_1|$. If the environmental bosons are in the same initial state and the system of interest is initially at the coherent state $|\alpha\rangle$, we find

$$t_R = \frac{1}{2g|\alpha|^2 + M\lambda\langle N_2\rangle}, \tag{24}$$

where $\langle N_2\rangle = \langle b_j^\dagger b_j\rangle = [\exp(\frac{\hbar\Omega}{k_B T}) - 1]^{-1}$ and $g \neq 0$. For sufficiently high temperatures, $t_R \approx 1/(\sqrt{M}\lambda\Delta_2)$; this depends basically on the effective Hilbert-space size and on the coupling strength λ, and differs from Eq. (8) mainly due to the factor $\sqrt{M}$. The difference between the characteristic times t_R and τ_R confirms previous results that indicate that the classical limit depends on the choice of the observable [12,33]. A different result can be found in Ref. [32], in a similar model; this suggests the inexistence of a general result as far as the existence of a single quantum break time is concerned. The revivals of the position operator mean value are signatures of quantum correlations. Since they vanish in the limit $\Delta_2 \rightarrow \infty$, a kind of classical limit [12,33] may be induced by the phase reservoir even with one degree of freedom.

IV. CONCLUSION

The analytical expressions for the decoherence time and the revival lifetime indicate that at short times and around the revival times the decoherence dynamics is determined by the coupling Hamiltonian and the variances of the initial states of the system and the environment. Far from these times, the linear entropy seems to be always near its maximum value of 1/2 (see Figs. 3 and 5). This suggests that for the model studied the most relevant features of decoherence process are determined by the coupling Hamiltonian and the initial state variances. Provided these parameters are fixed, the number of environmental degrees of freedom and the specific initial state on the environment seem to be unimportant: the way the entropy is produced by the environment seems to be settled by the effective Hilbert-space size. These conclusions depend crucially on the fact that we are dealing with a phase reservoir, but they do not depend on the choice of the quartic oscillator. Indeed, maintaining interaction Hamiltonian (6), the results do not change if we chose other Hamiltonians for the system and for the environment obeying commutation relations (2).

Our investigation suggests that the classical limit may be induced in the system of interest by the phase reservoir even with one degree of freedom. For Hamiltonian (12), the probability of observing the quantum coherence revivals vanishes for sufficiently large effective Hilbert-space sizes, as may be seen using the time-energy uncertainty relation. On the other hand, relevant quantum coherence revivals may be avoided in a long period if we choose interaction Hamiltonian (18), as it is shown in Fig. 5. Finally, the revivals of the expectation value of the position operator of a quartic oscillator, which is typical quantum phenomena, are retarded by the one degree of freedom phase reservoir.

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